In the figure shown, a long straight wire carries current $i_1$ and a rectangular loop carries current $i_2$. $i_1 = 30 \, \text{A}$, $i_2 = 20 \, \text{A}$, $a = 1.0 \, \text{cm}$, $b = 8.0 \, \text{cm}$, and $L = 30.0 \, \text{cm}$. In unit vector notation, what is the net force on the loop due to $i_1$?

**Solution:**

We will determine the magnetic field due to wire $i_1$, then use that field to find the force upon $i_2$.

First, use Ampere’s Law, with an Amperian loop of radius $r$ around $i_1$. Along this loop, the magnetic field points circumferentially, along the same direction as the tangent vector of the loop. So $\mathbf{B} \cdot d\mathbf{s} = B ds$ since the vectors are parallel. Furthermore, the magnetic field can be taken out of the integral since it has a constant magnitude everywhere on this loop. The integral is therefore trivial:

$$
\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I
$$

$$
B \oint ds = \mu_0 I
$$

$$
B 2\pi r = \mu_0 i_1
$$

$$
B = \frac{\mu_0 i_1}{2\pi r}
$$

The direction of this field is circumferential using the right hand rule, so in the region below $i_1$, it points *into* the page. Now, we need to use this field to find the force $F/l = i_2 \times B$ on each segment of the wire loop, $i_2$. As explained in the video, using the right-hand rule, we determine that the forces on the vertical segments points in opposite directions. These
forces are equal, because they are the same distance from the wire. The horizontal segments have opposite directions, but the force on the top segment is more, since \( r \) is smaller there, so \( B \) is greater. In particular, the force on the top segment is:

\[
\frac{|F_{\text{top}}|}{l} = i_2 B \sin \theta \\
\frac{|F_{\text{top}}|}{L} = i_2 B \\
\frac{|F_{\text{top}}|}{L} = i_2 \frac{\mu_0 i_1}{2\pi r} \\
\frac{|F_{\text{top}}|}{L} = i_2 \frac{\mu_0 i_1}{2\pi a}
\]

Plugging in the given values gives: \( F_{\text{top}} = 3.6 \text{ mN} \). Using the right hand rule, this force points upward, so \( F_{\text{top}} = 3.6 \text{j mN} \). Likewise, the force on the bottom segment has magnitude:

\[
\frac{|F_{\text{bot}}|}{l} = i_2 B \sin \theta \\
\frac{|F_{\text{bot}}|}{L} = i_2 \frac{\mu_0 i_1}{2\pi(a + b)}
\]

This gives \( F_{\text{bot}} = -0.4 \text{j mN} \). So the net force on the loop is \( F = 3.2 \text{j mN} \).