The figure shows a wooden cylinder with 10 turns of wire wrapped around it longitudinally. The mass of the cylinder is 0.25 kg, its longitudinal length is 10 cm. The cylinder is released on an inclined plane, of angle 15 degrees. If there is a vertical uniform magnetic field of magnitude 0.5 Teslas, what is the minimum current through the coil to keep the cylinder from rolling down the plane?

Solution:

The goal is to get the cylinder in equilibrium, so we must balance the forces and torques (because it is an extended body subject to rotation about its center of mass).

As noted in the video, we choose a coordinate system for the balance of the forces that points along the direction of the normal force (y) and parallel to the ramp (x). Since the gravitational force points along a skew direction to these axes, we can decompose this vector into its x-component ($mg\sin\theta$) and y-component ($mg\cos\theta$).

Therefore, balancing the forces gives:

\[ mg\sin\theta = F_f \]
\[ mg\cos\theta = F_n \]

As explained in the video, the gravitational force and normal force provide no torques, so we must only consider the frictional force, which provides a torque of:

\[ \tau_f = r \times F_f \]
\[ |\tau_f| = rF_f \]
\[ |\tau_f| = rmg\sin\theta \]

The second line follows because the angle between the force and radial vector is ninety degrees.
Finally, we need the torque provided by the magnetic field, which points in the opposing direction as the torque of friction as explained in the video. The magnitude of the magnetic torque is:

\[
|\tau_B| = \mu B \sin \theta_{\mu,B} \\
|\tau_B| = I_{\text{total}} AB \sin \theta_{\mu,B} \\
|\tau_B| = NI(2rL)B \sin \theta
\]

Here, \( N \) is the number of turns of wire in the loop. The angle between the magnetic moment and the magnetic field equals the angle of incline of the ramp, as shown in the video, so \( \sin \theta_{\mu,B} = \sin \theta \). The area of the current loop, \( A \), equals the area of the rectangle spanned by the wires, which is the diameter of the cylinder times its length, \( 2rL \).

Equating the magnetic and frictional torques gives:

\[
I = \frac{mg}{2LBN} = 2.45
\]

So the minimum required current is 2.45 Amps to hold the cylinder in equilibrium.