The figure gives the electric potential $V(x)$ along a copper wire carrying uniform current, from a point of higher potential $V_s = 12\mu V$ at $x = 0$ to a point of zero potential at $x_s = 3.00$ m. The wire has a radius of 2.00 mm. What is the current in the wire?

Solution:

We use the fact that:

$$J = \frac{E}{\rho}$$
$$I = \int JdA$$
$$E = -\frac{dV}{dx}$$

The first and last equations will allow us to use the information from the voltage graph to get the current density. Then, from this, we can use the second equation to get the total current; since current density will be uniform, the integral just gives $I = JA$.

Since voltage is linear, its spatial derivative equals the slope, so

$$E = -\frac{dV}{dx}$$
$$E = \frac{0 - 12\mu V}{3 - 0} \text{ m}$$
$$E = 4 \cdot 10^{-6} V/m$$

The resistivity of copper is $\rho_{Cu} = 1.69 \cdot 10^{-8} \Omega \cdot \text{ m}$, so the current density is:
\[ J = 237 \, A/m^2 \]

The cross sectional area of the wire is \( A = \pi (0.002)^2 = 1.26 \cdot 10^{-5} \). So the total current is:

\[
I = JA \\
I = 2.97 \, mA
\]